On Combining High and Low Q^2 Information on the Polarized Parton Densities

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Abstract

We draw attention to some problems in the combined use of high-Q^2 deep inelastic scattering (DIS) data and low-Q^2 hyperon β-decay data in the determination of the polarized parton densities. We explain why factorization schemes like the JET or AB schemes are the simplest in which to study the implications of the DIS parton densities for the physics of the low-Q^2 region.

1. Introduction

Our most precise knowledge of the internal partonic structure of the nucleon has come from decades of experiments on unpolarized Deep Inelastic Scattering (DIS) of leptons on nucleons. More recently there has been a dramatic improvement in the quality of the data on polarized DIS and consequently an impressive growth in the precision of our knowledge of the polarized parton densities in the nucleon. However, it will be a long time before the polarized data, for the moment limited to neutral current reactions, can match the unpolarized data in volume and accuracy. As a consequence, almost all analyses of the polarized parton densities supplement the DIS (large Q^2) data with information stemming from low-Q^2 weak interaction reactions. More specifically, it is conventional to use the values of G_A/G_V from neutron β-decay, and 3F-D from hyperon β-decays to help to pin down the values of the first moments of certain combinations of parton densities.

However, the standard way of doing this has been criticized because it essentially assumes exact $SU(3)_f$ flavor symmetry for the hyperon β-decays, whereas the growing precision of the measurements of magnetic moments and G_A/G_V ratios in hyperon semi-leptonic decays may be indicating a non- negligible breakdown of the flavor symmetry. Thus several attempts have been made to incorporate some symmetry breaking in the combined analysis of weak interaction data and polarized DIS data.

We wish to point out in this note that there are inconsistencies in some of the schemes and to draw attention to certain essential requirements in any attempts to include $SU(3)_f$ breaking in such combined analyses. We present also some comments regarding the question of the implications of the DIS parton densities for the physics of the low-Q^2 region.

2. Some consequences of scheme dependence

From the measured spin asymmetries $A_\parallel$ and $A_\perp$ in the inclusive DIS of leptons on nucleons one obtains information on the spin structure function $g_1(x,Q^2)$ of the nucleon. In the next to leading order (NLO) QCD approximation the quark-parton decomposition of $g_1(x,Q^2)$ has the following form:

$$g_1(x,Q^2) = \frac{1}{2}\sum_q^{N_f} e_q^2 \left[(\Delta q + \Delta\bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \delta C_G\right], \quad (1)$$

where $\Delta q(x,Q^2), \Delta\bar{q}(x,Q^2)$ and $\Delta G(x,Q^2)$ are quark, anti-quark and gluon polarized densities which evolve in Q^2 according to the spin-dependent NLO DGLAP equations

[1]. (Q^2 denotes the squared four-momentum of the exchanged virtual photon.) In (1) $\delta C_{q,G}$ are the NLO terms in the spin-dependent Wilson coefficient functions and the symbol $\otimes$ denotes the usual convolution in Bjorken x space. N_f is the number of flavors.

In order to link the information on the polarized parton densities obtained from the DIS data with the information from hyperon semi-leptonic weak decays it is convenient to rewrite (1) in terms of SU(3) flavor nonsinglet $\Delta q_{3,8}(x, Q^2)$ and singlet $\Delta\Sigma(x, Q^2)$ combinations of the quark densities ($N_f = 3$):

$$\begin{aligned} g_1^{p(n)}(x, Q^2) &= \frac{1}{9}[(\pm\frac{3}{4}\Delta q_3 + \frac{1}{4}\Delta q_8 + \Delta\Sigma) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) \\ &+ \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \delta C_G] \ , \end{aligned} \tag{2}$$

where

$$\Delta q_3(x, Q^2) = (\Delta u + \Delta\bar{u})(x, Q^2) - (\Delta d + \Delta\bar{d})(x, Q^2) \ , \tag{3}$$

$$\begin{aligned} \Delta q_8(x, Q^2) &= (\Delta u + \Delta\bar{u})(x, Q^2) + (\Delta d + \Delta\bar{d})(x, Q^2) \\ &- 2(\Delta s + \Delta\bar{s})(x, Q^2) \ , \end{aligned} \tag{4}$$

$$\Delta\Sigma(x, Q^2) = (\Delta u + \Delta\bar{u})(x, Q^2) + (\Delta d + \Delta\bar{d})(x, Q^2) + (\Delta s + \Delta\bar{s})(x, Q^2) \ . \tag{5}$$

Then for $\Gamma_1^{p(n)}(Q^2)$, the first moments of the proton and neutron structure functions $g_1^{p(n)}$, one has

$$\begin{aligned} \Gamma_1^{p(n)}(Q^2) &= \int_0^1 dx g_1^{p(n)}(x, Q^2) \\ &= \frac{1}{9}[\pm\frac{3}{4}a_3 + \frac{1}{4}a_8 + a_0(Q^2)](1 - \frac{\alpha_s(Q^2)}{\pi}) \ , \end{aligned} \tag{6}$$

where a_3 and a_8 are the nonsinglet axial charges corresponding to the $3^{\rm rd}$ and $8^{\rm th}$ components of the axial vector Cabibbo currents expressed in terms of the first moments of the quark densities (3) and (4) $[\Delta q(Q^2) \equiv \int_0^1 dx \Delta q(x, Q^2)]$

$$a_3 = (\Delta u + \Delta\bar{u})(Q^2) - (\Delta d + \Delta\bar{d})(Q^2) \ , \tag{7}$$

$$a_8 = (\Delta u + \Delta\bar{u})(Q^2) + (\Delta d + \Delta\bar{d})(Q^2) - 2(\Delta s + \Delta\bar{s})(Q^2) \ . \tag{8}$$

Note that while Δq and $\Delta\bar{q}$ depend on Q^2, a_3 and a_8 are conserved (Q^2 independent) quantities.

In (6) $a_0(Q^2)$ is the singlet axial charge, which depends on Q^2 because of the axial anomaly. It must be emphasized that the connection between $a_0(Q^2)$ and the factorization scheme dependent quantity $\Delta\Sigma$, the first moment of the singlet quark density

(5), is different for the various factorization schemes used for the QCD calculations of the structure function g_1.

So, in the $\overline{\rm MS}$ scheme

$$a_0(Q^2) = \Delta\Sigma(Q^2)_{\overline{\rm MS}} , \tag{9}$$

whereas for the JET and AB schemes in which $\Delta\Sigma$ is Q^2 independent,

$$a_0(Q^2) = \Delta\Sigma_{\rm JET(AB)} - N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2)_{\rm JET(AB)} . \tag{10}$$

As pointed out in [2] one can actually define a family of such schemes. Among them the most popular are the so-called AB (Adler-Bardeen) [3] and JET (see [4] and references therein) schemes. In (10) ΔG is the first moment of the polarized gluon density.

For the further considerations it is useful to recall the transformation rule connecting the first moments of the strange sea quarks in the nucleon, $(\Delta s + \Delta \bar{s})$, in the $\overline{\rm MS}$ and JET(AB) schemes:

$$(\Delta s + \Delta \bar{s})_{\rm JET(AB)} = (\Delta s + \Delta \bar{s})(Q^2)_{\overline{\rm MS}} + \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2)_{\overline{\rm MS}} . \tag{11}$$

Note that the LHS of (11) is Q^2 independent.

It is important to mention here that the difference between the values of $\Delta\Sigma$ or of $(\Delta s + \Delta \bar{s})$, obtained in the $\overline{\rm MS}$ and JET(AB) schemes could be large due to the axial anomaly. Indeed, as a consequence of the anomaly, the term $\alpha_s \Delta G$ in (10) and (11) behaves as [5]:

$$\alpha_s(Q^2)\Delta G(Q^2) = {\rm const} + \mathcal{O}(\alpha_{\rm s}({\rm Q}^2)) , \tag{12}$$

i.e., it is not really of order α_s. To illustrate how large the difference can be, we present the values of $(\Delta s + \Delta \bar{s})$ at $Q^2 = 1\ GeV^2$ obtained in our recent analysis [6] of the world DIS data in the $\overline{\rm MS}$ and JET(AB) schemes:

$$(\Delta s + \Delta \bar{s})_{\overline{\rm MS}} = -0.10 \pm 0.01, \qquad (\Delta s + \Delta \bar{s})_{\rm JET(AB)} = -0.06 \pm 0.01 . \tag{13}$$

3. What can be deduced in principle from DIS

As was shown in [7] if the DIS data on the independent structure functions g_1^p and $g_1^n(g_1^d)$ were perfect and QCD was the correct theory of the strong interactions, the individual parton densities

$$(\Delta u + \Delta \bar{u})(x, Q^2), \qquad (\Delta d + \Delta \bar{d})(x, Q^2), \qquad (\Delta s + \Delta \bar{s})(x, Q^2) \tag{14}$$

and $\Delta G(x,Q^2)$ or, equivalently, $\Delta q_{3,8}(x,Q^2), \Delta\Sigma(x,Q^2)$ and $\Delta G(x,Q^2)$ would be *uniquely* determined at some arbitrary $Q^2 = Q_0^2$. This follows from the fact that Δq_3 is fixed by the difference $g_1^p - g_1^n$ while the rest $\Delta q_8, \Delta\Sigma$ and ΔG can be determined separately from $g_1^p + g_1^n$ because of their *different* Q^2 evolution.

It is immediately clear, given the limited range of Q^2 available and the fact that the data are *not* perfect and have errors, that the separation of Δq_8, $\Delta\Sigma$ and ΔG from each other will not be very clear-cut. Nonetheless, *in principle*, the data fix $\Delta q_{3,8}(x,Q^2)$, $\Delta\Sigma(x,Q^2)$ and $\Delta G(x,Q^2)$ or, equivalently, via Eqs. (3), (4), and (5), $(\Delta u + \Delta\bar{u})(x,Q^2)$, $(\Delta d + \Delta\bar{d})(x,Q^2)$, $(\Delta s + \Delta\bar{s})(x,Q^2)$ and $\Delta G(x,Q^2)$.

It is also clear from the above that whereas the strange sea density $(\Delta s + \Delta\bar{s})$ is, in principle, *fixed* by the inclusive (electromagnetic current) data, these data give no information about the other sea quark densities in the nucleon, $\Delta\bar{u}$ and $\Delta\bar{d}$, and therefore, about the valence parts Δq_v of the quark densities. In order to extract them from the data (they are needed to make predictions for other processes, like polarized *pp* reactions, etc.), additional assumptions about the flavor decomposition of the sea are necessary. Conventionally, the following assumption has been used in most of the analyses

$$\Delta\bar{u} = \Delta\bar{d} = \lambda\Delta\bar{s} \ , \tag{15}$$

where λ is a parameter.

Given that the data fix $\Delta q_{3,8}$, $\Delta\Sigma$ and ΔG and that

$$(\Delta s + \Delta\bar{s})(x,Q^2) = \frac{1}{3}[\Delta\Sigma(x,Q^2) - \Delta q_8(x,Q^2)] \ , \tag{16}$$

we see that while $\Delta\bar{u}(\Delta u_v)$ and $\Delta\bar{d}(\Delta d_v)$ are sensitive to the assumptions about the flavor decomposition of the sea, the result for $(\Delta s+\Delta\bar{s})(x,Q^2)$ as well as for $\Delta G(x,Q^2)$ should *not* change as λ is varied. This provides a serious test for the stability of any analysis and was confirmed numerically in our study [7]. (Note that the attempts [8] to extract the valence quarks from *semi-inclusive* data without assumptions about the sea are not entirely successful because of the quality of these data at present.)

In other words, inclusive DIS data do not enable us to test if the SU(3) symmetry of the sea is broken or not. What follows from these data and QCD is that $(\Delta s + \Delta\bar{s})(x,Q^2)$, the strange sea of the nucleon, does *not* depend on the symmetry breaking of the sea and therefore, only models, in which $(\Delta s + \Delta\bar{s})(x,Q^2)$ is *insensitive* to this breaking, are acceptable.

4. What we know about the partonic spin content of the nucleon from weak semi-leptonic hyperon decays

In addition to the information on the polarized parton densities from the DIS experiments very useful knowledge of their first moments comes from the hyperon semi-leptonic decays.

The Bjorken sum rule [9] tell us that

$$a_3 = (\Delta u + \Delta\bar{u})(Q^2) - (\Delta d + \Delta\bar{d})(Q^2) = \frac{G_A}{G_V}(n \to p) \equiv g_A \ , \tag{17}$$

where g_A is the neutron weak β-decay constant [10]:

$$g_A = 1.2601 \pm 0.0025 \ . \tag{18}$$

This sum rule reflects the isospin SU(2) symmetry which is well established. Assuming the usual SU(3) transformation properties of the axial currents and that the hyperons form an SU(3) octet, the hyperon β-decays fix a_8, the first moment of $\Delta q_8(x, Q^2)$, to be:

$$a_8 = (\Delta u + \Delta\bar{u})(Q^2) + (\Delta d + \Delta\bar{d})(Q^2) - 2(\Delta s + \Delta\bar{s})(Q^2) = 3\mathrm{F} - \mathrm{D} \ , \tag{19}$$

where

$$3\mathrm{F} - \mathrm{D} = 0.597 \pm 0.025 \quad [11]. \tag{20}$$

Depending on the data included in the hyperon β-decays analysis this value changes slightly: 0.601 ± 0.038 in [12] and 0.597 ± 0.019 in [13]. However, the large value of χ^2/DOF of the SU(3) symmetric fit (2.7 in [12] and 2.3 in [13]) is some evidence for SU(3) breaking. The issue of this breaking is treated in different models, which predict for a_8 values between 0.36 [12] and 0.85 [14]. The current KTeV experiment in Fermilab on the Ξ^0 β-decay, $\Xi^0 \to \Sigma^+ e\bar{\nu}$, (see [15] and references therein) will be a crucial test for them.

Given that because of the quality of the present DIS data the relations (17) and (19) have been used in addition in most of the analyses, it is quite important to understand what is the proper value of a_8.

In this connection let us consider the SU(3) symmetry breaking model suggested in Ref. [16]. In this model a specific breaking of the symmetry in the hyperon decays is introduced by treating the wave functions of the octet of baryons as made up of a valence quark part, which is $\mathrm{SU(3)_f}$ symmetric, and a sea part piece, which is allowed to break the symmetry. As a result of a combined analysis of the DIS and hyperon decay data it has been found [17] that the strange content of the proton, $(\Delta s+\Delta\bar{s})(Q^2)$, is sensitive to the SU(3) symmetry breaking of the *sea*, in contradiction with the fact stressed in Section 3 that, at least in principle, the DIS data alone fix the value of $(\Delta s+\Delta\bar{s})(Q^2)$.

A possible origin of this discrepancy may be seen in the following. An essential result of this model is the replacement of the $\mathrm{SU(3)_f}$ result (19) by the modified form

$$a_8 = \mathrm{3F - D} + 2\epsilon(\mathrm{\Delta s + \Delta\bar{s}})(\mathrm{Q}^2)\ , \tag{21}$$

where $\epsilon = \lambda - 1$ measures the breaking of the symmetry.

Now we wish to point out that (21) is inconsistent. The inconsistency stems from the fact that in this model the first moment $\Gamma_1^{p(n)}(q^2)$, eq. (6), does not contain a contribution from the gluon. In order words, $a_0(Q^2)$ has the form given in (9), so that the analysis is effectively being carried out in an $\overline{\mathrm{MS}}$-type scheme. In such schemes, as is clear from (11), and is emphasized in (21), $(\Delta s+\Delta\bar{s})(Q^2)$ varies with Q^2. Hence (21) is inconsistent, since in the $\overline{\mathrm{MS}}$ scheme the LHS is independent of Q^2.

Note that although a_8 cannot be well fixed at present using the DIS data alone, one should not consider SU(3) breaking models in which a_8 and $(\Delta s+\Delta\bar{s})$ are dependent. Such models are in contradiction with what follows for these quantities from QCD in the DIS region ($\Delta q_8(x,Q^2)$ and $(\Delta s+\Delta\bar{s})(x,Q^2)$ are independent due to the different Q^2 evolution).

5. How to link low Q^2 data with polarized DIS experiments

It is clear from the above that the polarized densities cannot be extracted well enough at present without linking the information from both high and low-Q^2 regions. (In future more accurate both inclusive and semi-inclusive DIS data would help us to extract a_8 independently and thus to test models of the SU(3) flavor symmetry breaking. It also appears that it might be possible in the not too distant future to do high intensity neutrino experiments with a polarized target.)

In this Section we present some comments regarding the combined use of DIS and low-Q^2 data and the question of the implications of the DIS parton densities for the physics of the low-Q^2 region.

i) It is simplest to deal with quantities independent of Q^2. While a_3 and a_8 are independent of Q^2, the singlet combination of quarks $\Delta\Sigma(Q^2)$, as well as $(\Delta s+\Delta\bar{s})(Q^2)$, are in general Q^2 dependent. However, there are factorization schemes like the AB and JET schemes, in which $\Delta\Sigma(Q^2)$ and $(\Delta s+\Delta\bar{s})(Q^2)$ do *not* depend on Q^2. Although the theoretical results for the physical quantities such as the polarized structure functions $g_1^{p(n)}$ are scheme independent, it is clear that only in schemes like AB and JET, is it meaningful to directly interpret $\Delta\Sigma$ as the contribution of the quark spins to the nucleon spin and to confront its value obtained from the high energy region with predictions from low energy models like constituent quark, chiral quarks models, etc.

ii) An important result from the present DIS data is that the singlet axial charge is small:$a_0(Q^2) \approx 0.2 - 0.3$ in the high energy region $Q^2_{\rm DIS} \geq 1\ GeV^2$. As a consequence, $(\Delta s + \Delta\bar{s})(Q^2)_{\overline{\rm MS}}$ in the $\overline{\rm MS}$ scheme is relatively large and negative in this region. This does not mean that $(\Delta s + \Delta\bar{s})(Q^2)_{\overline{\rm MS}}$ is large in the nonperturbative, low-Q^2 region too. To determine $(\Delta s + \Delta\bar{s})(Q^2)$ in the low-Q^2 region using the information on $a_0(Q^2)$ obtained in the DIS range, it is simplest to work in the JET(AB) scheme, in which this quantity is Q^2 independent. As seen from Eqs. (11) and (13), $(\Delta s + \Delta\bar{s})$ could be small, even zero (as expected in constituent quark models), depending on the sign and the size of the gluon polarization in the DIS region. Although $\Delta G(Q^2)$ is not well determined from the present data there is significant evidence from the experimental data, directly [18] and indirectly via Q^2 evolution effects (see, for example [6], [19]), that $\Delta G(Q^2)$ is positive and not too small: $\Delta G(Q^2) \sim 0.5 - 1.0$ at $Q^2 \sim 1\ GeV^2$.

Here we would like to recall the interesting possibility of obtaining independent information on the strange sea quarks from the *elastic* $\nu(\bar{\nu})N$ scattering. As shown in the papers [20], measurements on the $\nu(\bar{\nu})$ asymmetry of these reactions will allow to extract $(\Delta s + \Delta\bar{s})(Q^2)$ at $Q^2 \approx 0$ in a model-independent way.

iii) SU(3) symmetry breaking models invoked to explain hyperon decay data in the low-Q^2 range have to be consistent with the polarized parton densities and the QCD factorization scheme used in their determination from the DIS data.

6. Summary

We have presented a critical assessment of what can be learned at present about the partonic spin content of the nucleon from both low energy hyperon β-decays and polarized DIS data. It was pointed out that the simplest way to link consistently the theoretical and experimental results obtained in these different regions is to use for the calculation of the spin-dependent structure function g_1 factorization schemes like the JET and AB schemes. In these schemes, in addition to a_3 and a_8, the scheme dependent quantities $\Delta\Sigma$, $(\Delta s + \Delta\bar{s})$, the first moments of the singlet and strange sea densities, are also Q^2 *independent*. In the JET and AB schemes is it meaningful to interpret $\Delta\Sigma$ as the contribution of the quark spins to the nucleon spin and to compare its value obtained from DIS region with the predictions of the different (constituent, chiral, etc.) quark models at low Q^2. Finally, in such schemes the role of the gluon polarization in the high energy polarized experiments is more transparent.

Acknowledgments

One of us (D. S.) thanks S. B. Gerasimov for useful discussions concerning the hyperon β-decays. D. S. is grateful for the hospitality of the High Energy Section of the Abdus Salam International Centre for Theoretical Physics, Trieste, where this work has been completed. This research was partly supported by a UK Royal Society Collaborative Grant and by the Bulgarian National Science Foundation.

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